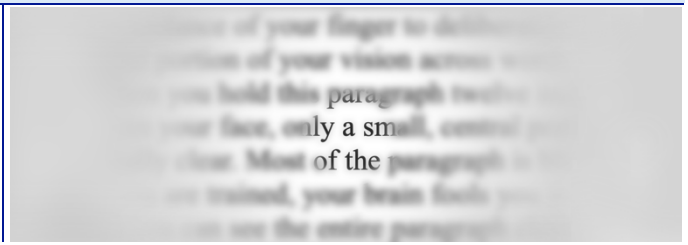
Beginner's Pre-Algebra
Careful reading and graphical reasoning

Table of Contents

Use this packet to develop reading and reasoning skills that are useful during Pre-Algebra. Throughout this document, "read" means pointing and reading aloud:

Point as you read aloud 指差喚呼

Use the guidance of your finger to deliberately drag the central portion of your vision across words as you read. When you hold this paragraph twelve inches away from your face, only a small, central portion of it is actually clear. Most of the paragraph is blurry. Unless you are trained, your brain fools you into thinking you can see the entire paragraph clearly.



Convert words into a diagram

Translate the following passage into pictures or tables that are easy for people who don't read English to understand.

Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?

Reading steps

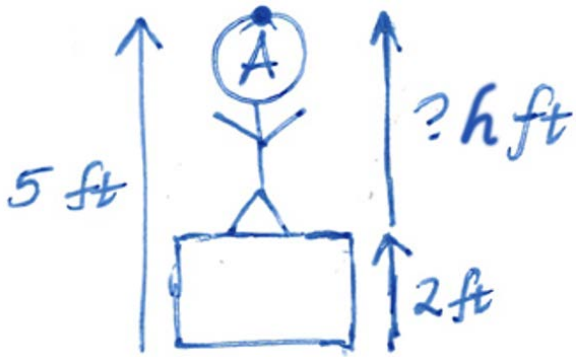
1. **Read** a short phrase containing one of the following items (occasionally, a phrase has multiple roles).
   a. **Object/person** (possibly with specification of characteristics)
   b. **Action** (possibly with specification of styles/manners in which action is carried out)
   c. **Location/time**
   d. **Quantity** (dimension or count)
2. **Draw** a simple representation of the phrase (unless the meaning of the phrase has already been sketched).
3. **Underline** the phrase. If no additional sketching was needed to illustrate the phrase, dash-underline the phrase.
4. Analyze **next** phrase.

Table 1. Breaking passage into short phrases

	Phrase	Main features	Sketch
1.	Alice	Object/person	Stick figure; capital "A" in face
2.	stands	Action	No additional sketching needed: Alice's stick figure was already drawn upright.
3.	on a box	Location/time	Rectangle immediately under Alice
4.	that has a height of 2 feet.	Quantity	Upward arrow from bottom to top of rectangle labeled "2 ft"
5.	The top of Alice's head	Object/person	Dot marking top of Alice's stick figure
6.	is 5 feet above the ground.	Quantity and Location/time	Upward arrow from bottom of rectangle to dot marking top of Alice labeled "5 ft"
7.	How tall is Alice?	Quantity (requested)	Upward arrow from bottom of Alice to dot marking top of Alice labeled "? *h* ft"

Showing the table above is usually unnecessary. Underlining the printed problem statement and drawing a sketch usually suffices.

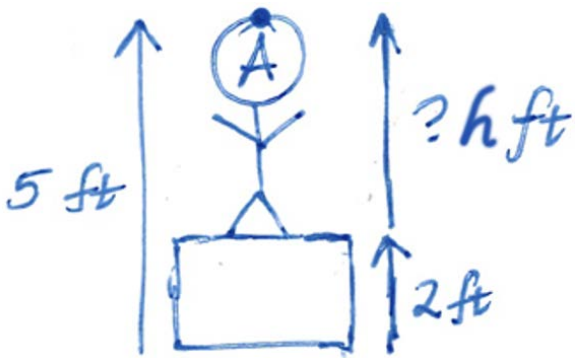
Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?



Convert a diagram into an equation and solve

The problem statement and sketch from the previous page are copied below and to the right.

> Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?



Answer the question.

Steps for converting a word problem's diagram into an equation and solving

1. Look for a starting equation.
    a. If there's a flashcard with a relevant equation, check that conditions the flashcard lists are met. If so, copy the equation.
    b. If no suitable flashcard equation is available, create a starting equation.
        i. Use your finger to point at and use your words to narrate a collection of related diagrammed quantities (Table 2, columns A and B).
        ii. Identify quantities, operations, and relations in your narration (Table 2, columns B, C, and D).
        iii. Algebraically abbreviate each quantity, operation, and relation (Table 2, columns C, D, and E).
2. Use gridded representations of pre-algebra (see page 5) to isolate the variable whose value is to be found.
3. Write a sentence presenting the value you found, with appropriate sign ($\pm$) and units.

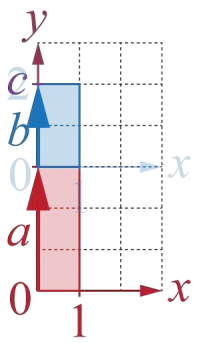
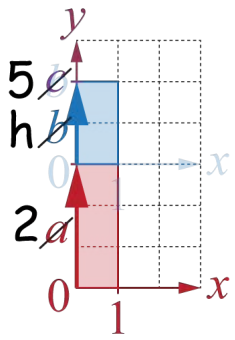
Table 2. Converting a diagram into an equation and solution

A. Gesture	B. Narrate	C. Quantities, operations, and relations	D. Category	E. Algebra
Trace finger up across arrow indicating the box's height	"Ascending through the 2-foot height of the box"	"Ascending through the 2-foot height of the box"	Quantity	2
Trace finger up across arrow indicating Alice's height	"and then further ascending through Alice's unknown height h"	"and then further ascending through"	Operation	$+$
		"Alice's unknown height h"	Quantity	h
Trace finger up across arrow from bottom of box to top of Alice's head	"travels through the same height change as ascending 5 feet from the bottom of the box to the top of Alice's head."	"travels through the same"	Relation	$=$
		"height change as ascending 5 feet from the bottom of the box to the top of Alice's head."	Quantity	5

Starting equation: $2 + h = 5$

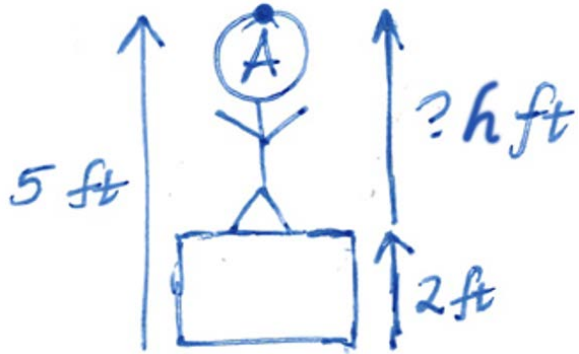
See pre-algebraic solution and summary sentence on next page.

Table 3. Algebraically solve (see page 5 for templates):

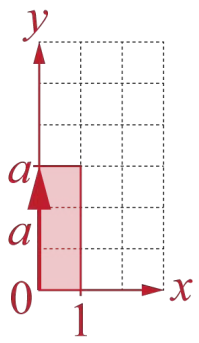
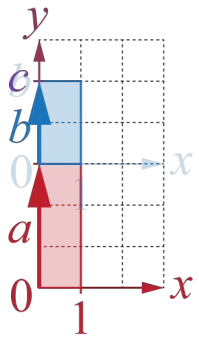
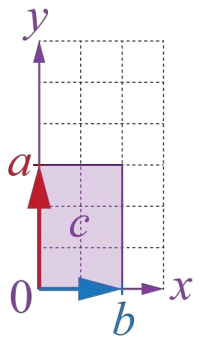
A. Work people show	B. Algebraic work	C. Gridded work	D. Explanation
$2 + h = 5$			
	$a + b = c$		$2 + h = 5$ has the form $a + b = c$, so copy the algebraic and gridded templates for B. Addition. Since the gridded template for B. Addition is also the gridded template for C. Subtraction, copy the algebraic template for C. Subtraction too.
	$b = c - a$		
	$2 + h = 5$		Replace placeholder letters with symbols specific to current problem. It's OK if, as in this example, replacing placeholder letters reveals that your gridded template isn't drawn to scale for your problem.
	$h = 5 - 2$		
$h = 5 - 2$			Copy version of algebraic subtraction template customized with symbols from current problem.
$h = 3$			Using a calculator to evaluate $5 - 2$ is fine.

Summarize: "Alice's height is 3 feet."

(If you sketched the situation to scale, you could also have immediately read Alice's height off of the drawing).



Pre-algebra templates

	Name	Numeric example	Template		Gridding steps
			Algebraic	Gridded	
A.	**(Real) number**	3	a		1. Draw vertical arrow from origin (0) to y-tick labeled a. Label arrow a. 2. Regard vertical arrow as left edge of a rectangle of width 1. 3. Vertical arrow and rectangle represent the number a individually and together.
B.	**Addition**	$3 + 2 = 5$	$a + b = c$		1. Represent the number a. 2. Regard arrowhead of arrow for a as origin of a new axis system. 3. Use new axis system to represent the number b. 4. Use original axes to read off y-tick, c, of arrowhead you just drew and complete sketch of the number c.
C.	**Subtraction**	$2 = 5 - 3$	$b = c - a$		
E.	**Multiplication**	$(3)(2) = 6$	$(a)(b) = c$		1. Represent number a. Ignore rectangle, if desired. 2. Draw horizontal arrow from origin (0) to x-tick labeled b. 3. Draw rectangle using vertical arrow for a and horizontal arrow for b as sides. Label signed number of unit squares, c, rectangle covers. 4. A rectangle in Quadrant I or Quadrant III has a positive signed number of unit squares. A rectangle in Quadrant II or Quadrant IV has a negative signed number of unit squares.
F.	**Division**	$2 = \frac{6}{3}$	$b = \frac{c}{a}, a \neq 0$		